

# NEUTRINO NSI, GLOBAL ANALYSES AND COPULAS

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*NF03 Working Group  
September 17, 2020*

# GLOBAL ANALYSES WITH NEUTRINO NSI

- ▶  $\mathcal{L} \supset -2\sqrt{2}G_F \sum_{f,\alpha,\beta} [\epsilon_{\alpha\beta}^{f,L}(\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta)(\bar{f} \gamma_\mu P_L f) + \epsilon_{\alpha\beta}^{f,R}(\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta)(\bar{f} \gamma_\mu P_R f)]$
- ▶  $f = e, u, d$
- ▶  $\alpha, \beta = \{e, \mu, \tau\}$
- ▶  $3 \times 6 \times 2 = 36$  real-valued NSI
- ▶ We combined CE $\nu$ NS, E $\nu$ ES, and Oscillation experiments in a global analysis to help resolve the many known degeneracies between these NSI

**A global analysis strategy to resolve neutrino NSI degeneracies with scattering and oscillation data**

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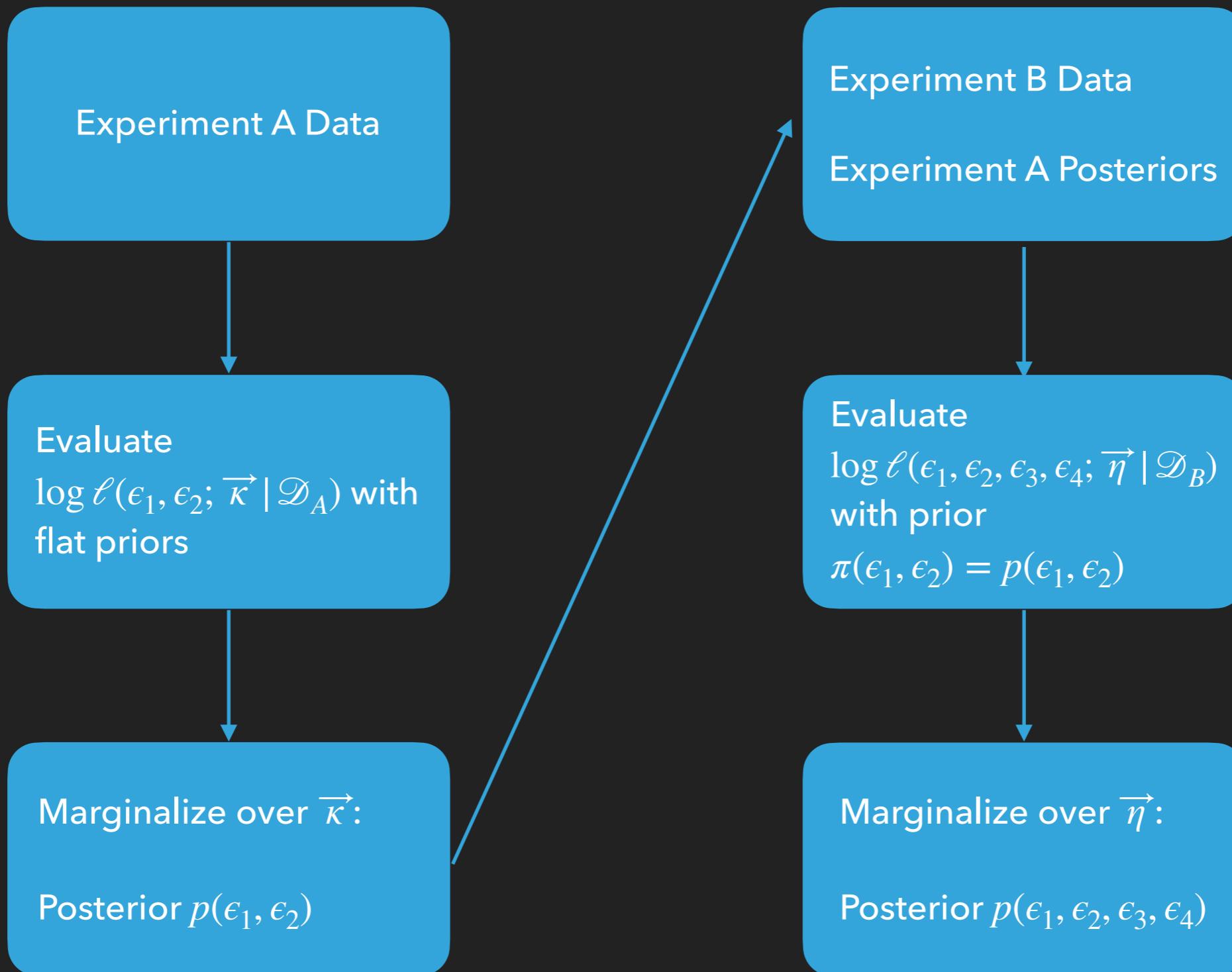
**ABSTRACT:** Neutrino non-standard interactions (NSI) with the first generation of standard model fermions can span a parameter space of large dimension and exhibit degeneracies that cannot be broken by a single class of experiment. Oscillation experiments, together with neutrino scattering experiments, can merge their observations into a highly informational dataset to combat this problem. We consider combining neutrino-electron and neutrino-nucleus scattering data from the Borexino and COHERENT experiments, including a projection for the upcoming coherent neutrino scattering measurement at the CENNS-10 liquid argon detector. We extend the reach of these datasets over the NSI parameter space with projections for neutrino scattering at a future multi-ton scale dark matter detector and future oscillation measurements from atmospheric neutrinos at the Deep Underground Neutrino Experiment (DUNE). In order to perform this global analysis, we adopt a novel approach using the copula method, utilized to combine posterior information from different experiments with a large, generalized set of NSI parameters. We find that the contributions from DUNE and a dark matter detector to the Borexino and COHERENT fits can improve constraints on the electron and quark NSI parameters by up to a factor of 2 to 3, even when relatively many NSI parameters are left free to vary in the analysis.

[arXiv: 2002.03066](https://arxiv.org/abs/2002.03066)

## A HYPOTHETICAL SCENARIO

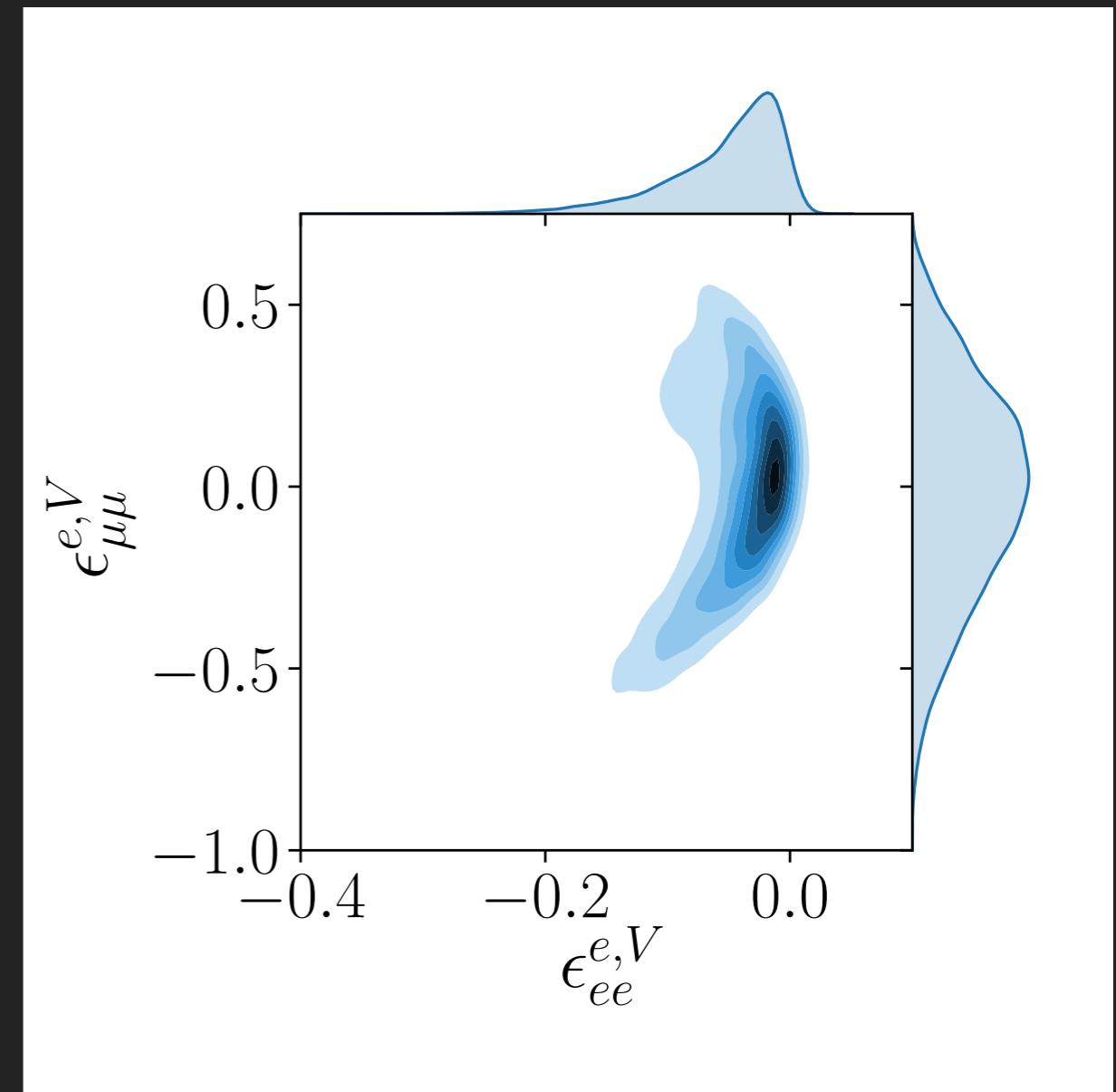
- ▶ We wish to perform a global analysis, combining...
- ▶ Experiment A Data
  - ▶ 2 new physics parameters  $\epsilon_1, \epsilon_2$
  - ▶ 4 systematics/statistical nuisance parameters  $\kappa_1, \kappa_2, \kappa_3, \kappa_4$
- ▶ Experiment B Data
  - ▶ 4 new physics parameters  $\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4$
  - ▶ 5 systematics/statistical nuisance parameters  $\eta_1, \eta_2, \eta_3, \eta_4, \eta_5$
- ▶ One possible solution: construct a global likelihood function
  - ▶  $\mathcal{L} = \mathcal{L}(\vec{\epsilon}; \vec{\kappa}, \vec{\eta})$
  - ▶ This can be computationally expensive

# BAYESIAN STRATEGY: “PRIOR-FLOW”



# HOW DO WE MODEL A JOINT POSTERIOR DISTRIBUTION?

- ▶ In order to use the full joint posterior  $p(\epsilon_1, \epsilon_2)$  as a prior in a Bayesian sampler, we need to know how to model the distribution
- ▶ Posteriors on model parameters can be non-trivial, cannot be fit by functions of closed form
- ▶ We need other technology: the Copula



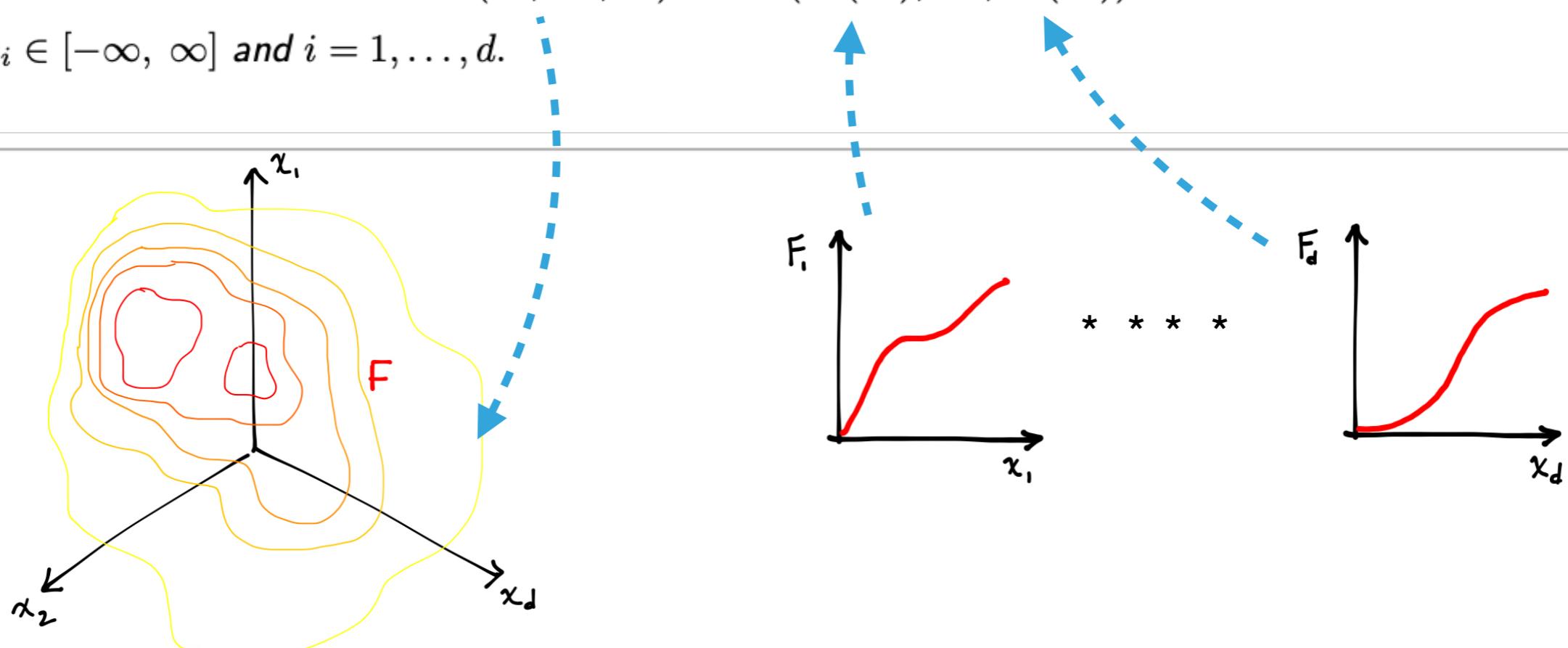
# THE COPULA AND SKLAR'S THEOREM

**Definition 1** A  $d$ -dimensional copula,  $C : [0, 1]^d \rightarrow [0, 1]$  is a cumulative distribution function (CDF) with uniform marginals.

**Theorem 2 (Sklar's Theorem 1959)** Consider a  $d$ -dimensional CDF,  $F$ , with marginals  $F_1, \dots, F_d$ . Then there exists a copula,  $C$ , such that

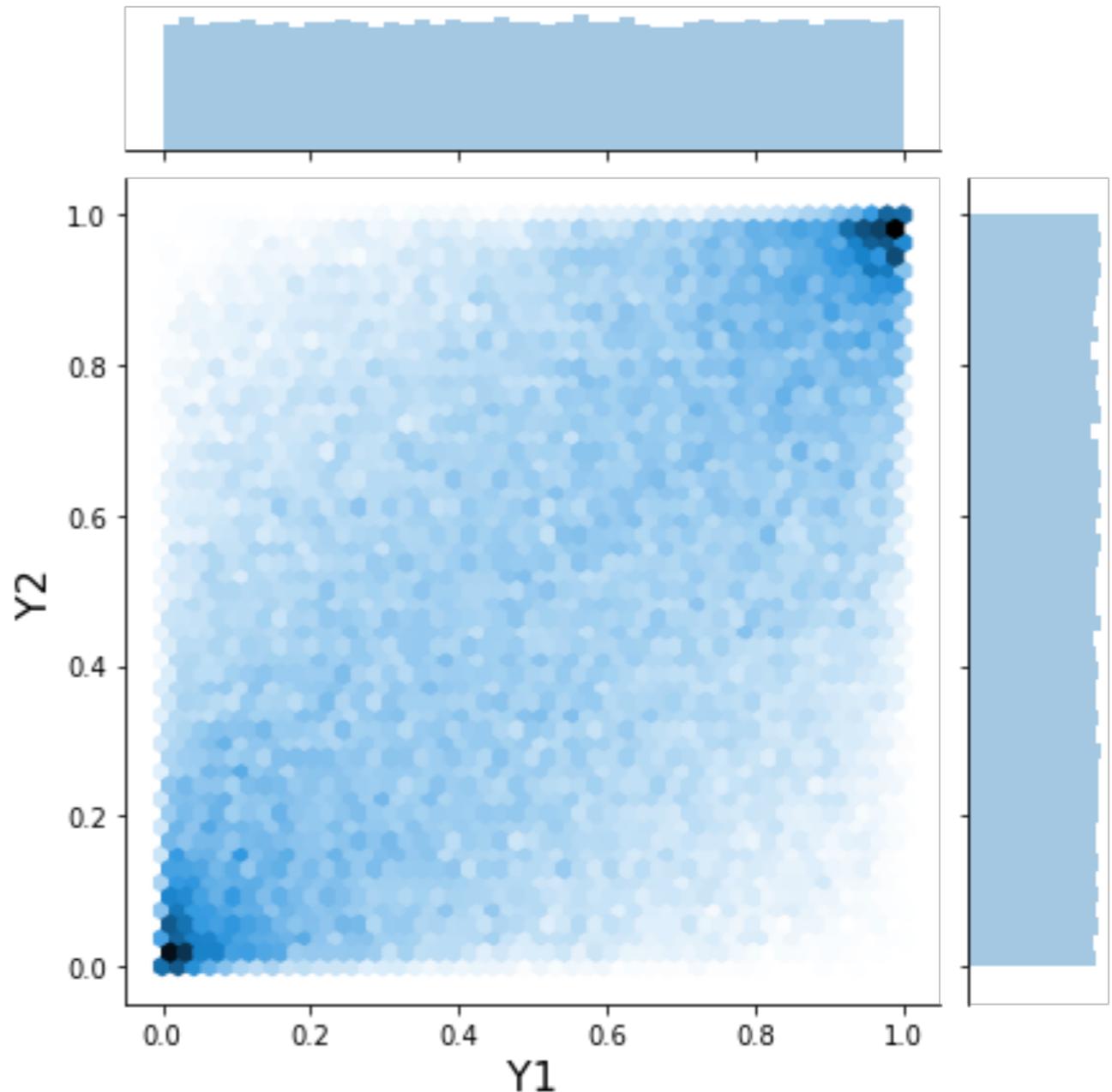
$$F(x_1, \dots, x_d) = C(F_1(x_1), \dots, F_d(x_d)) \quad (2)$$

for all  $x_i \in [-\infty, \infty]$  and  $i = 1, \dots, d$ .

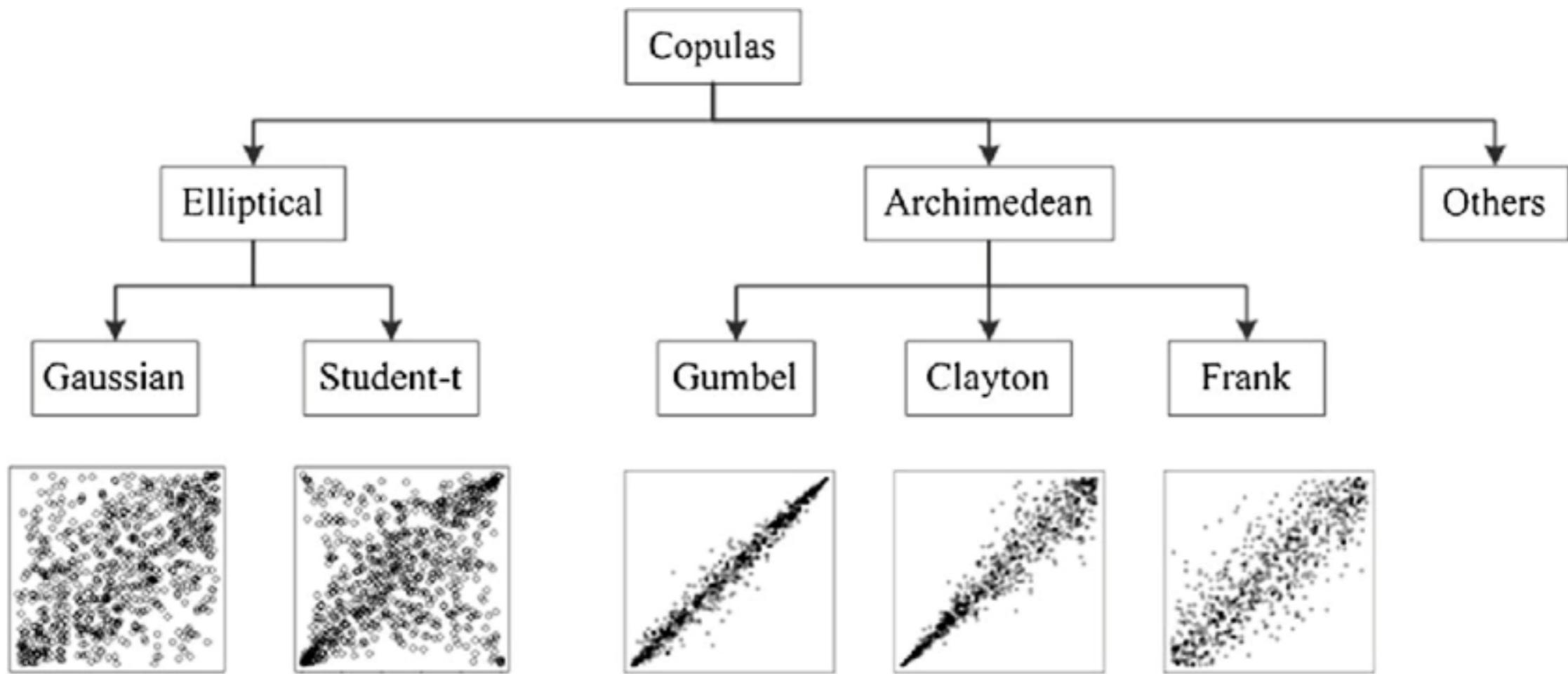


## COPULAS PROVIDE A DEPENDENCY STRUCTURE

- ▶ Copulas have flat marginals
- ▶ All their information is contained in the joint
- ▶ We use Sklar's theorem to pass in the Marginals of a distribution to a Copula fit to the joint of the distribution

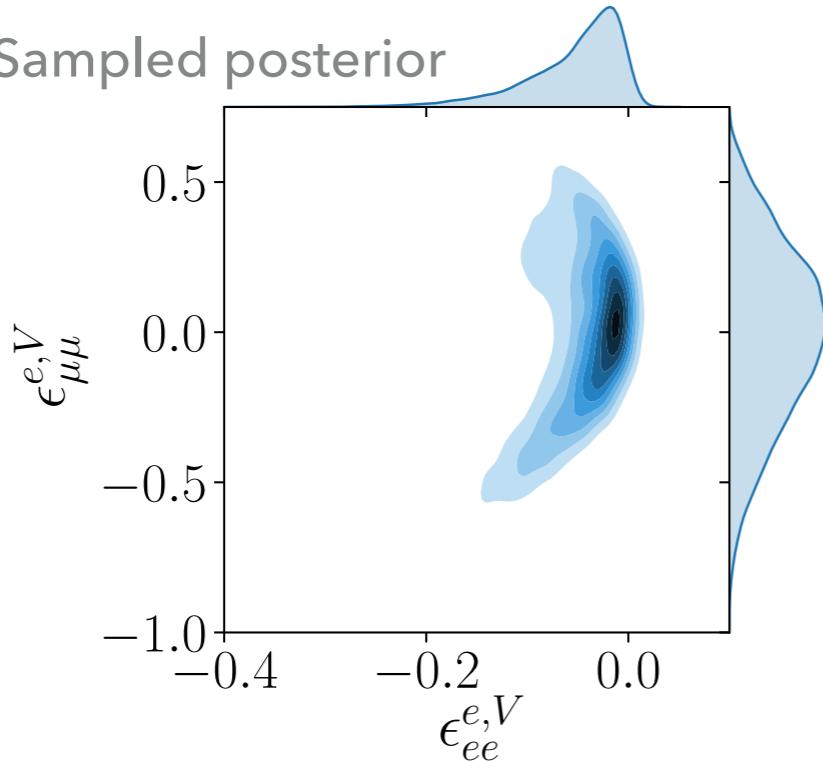


# FAMILIES OF COPULAS

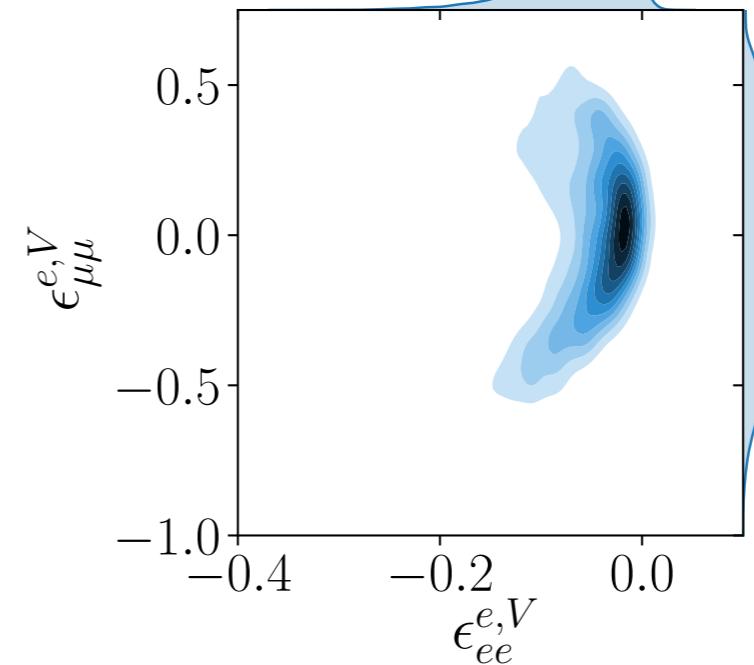


*On the aggregation of credit, market and operational risks - Li, et al (2015)*

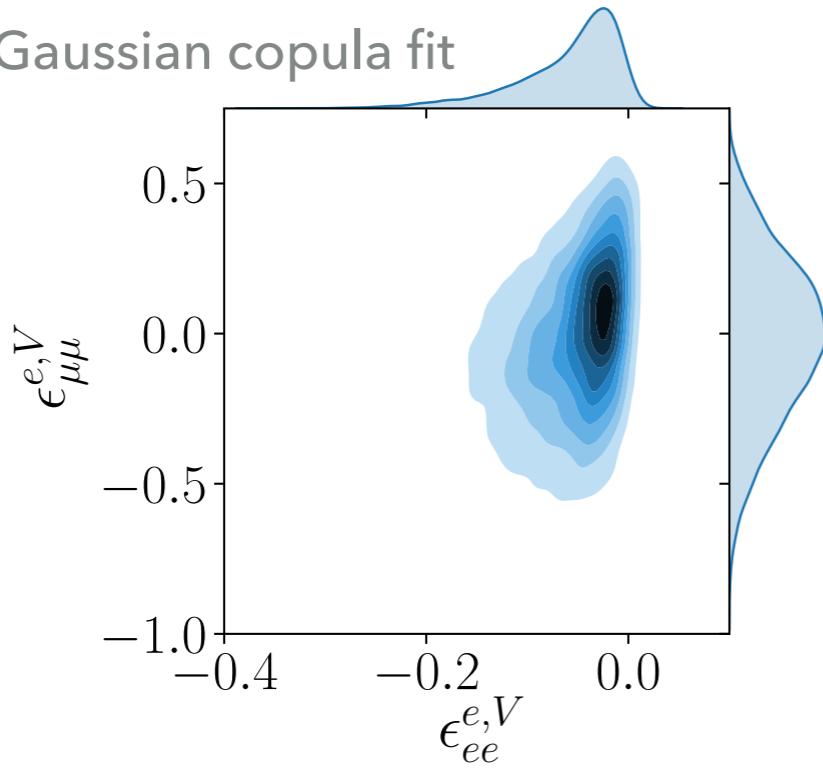
Sampled posterior



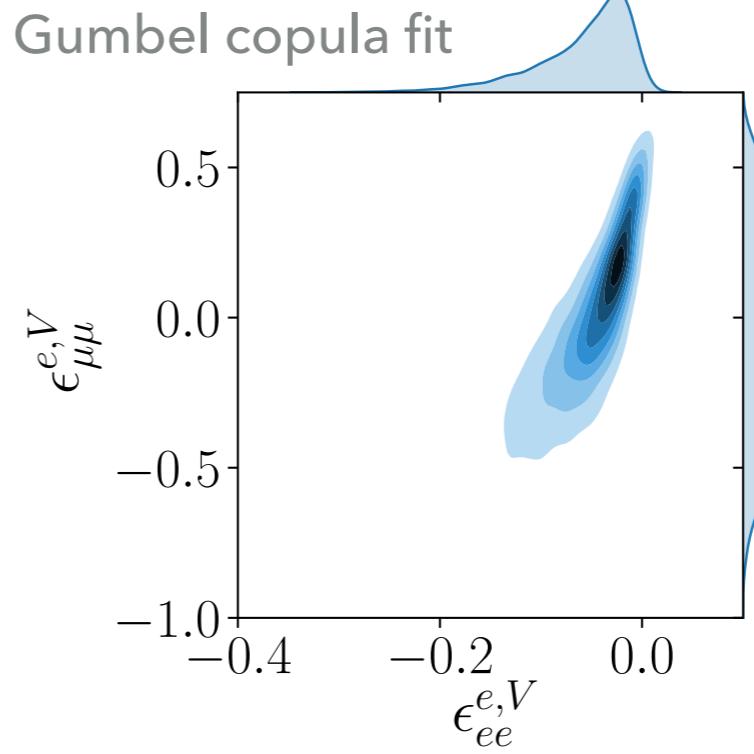
Empirical copula fit



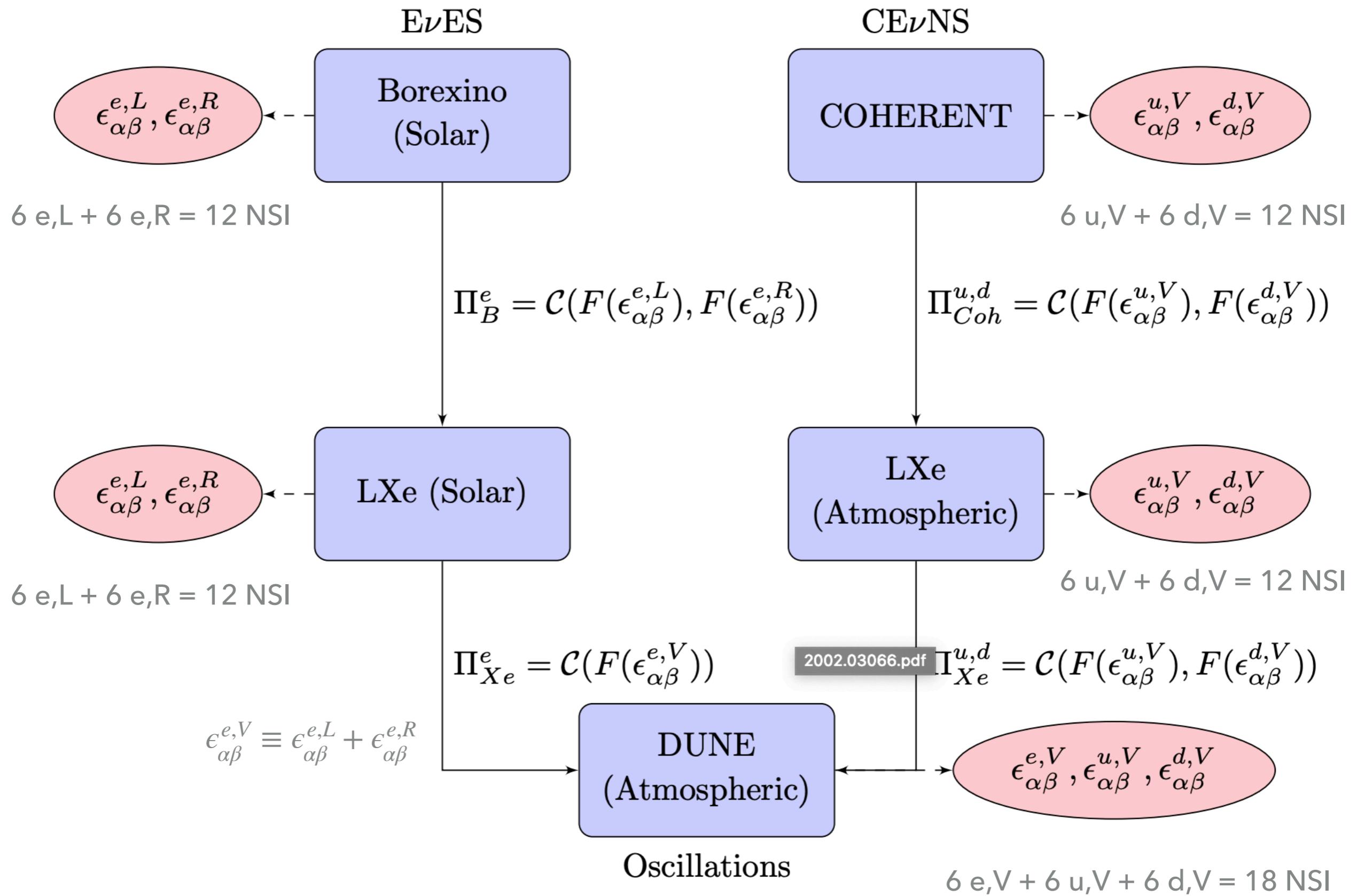
Gaussian copula fit

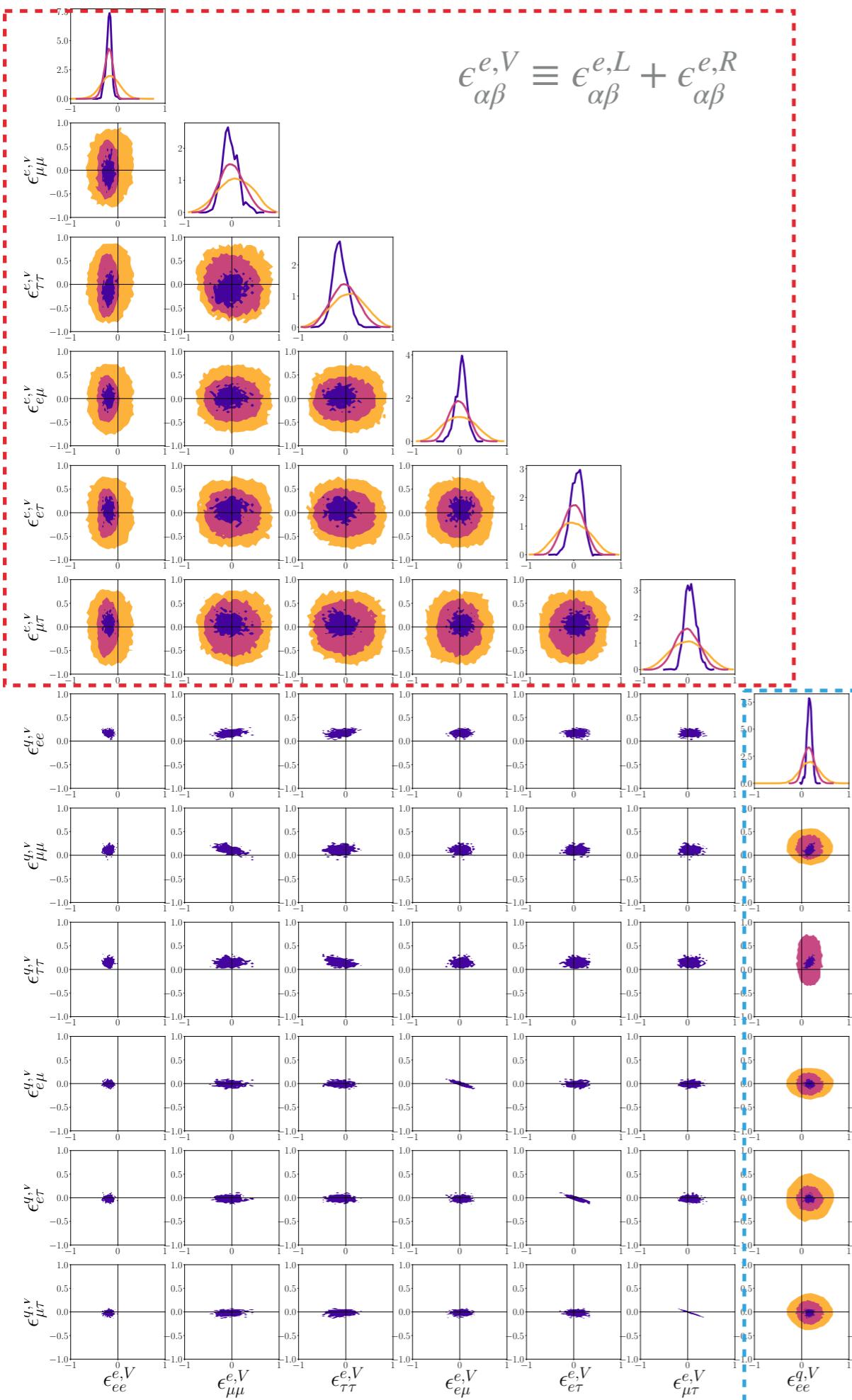


Gumbel copula fit



- ▶ We fit an Empirical copula to the posterior samples, avoiding biases of the specific copula families



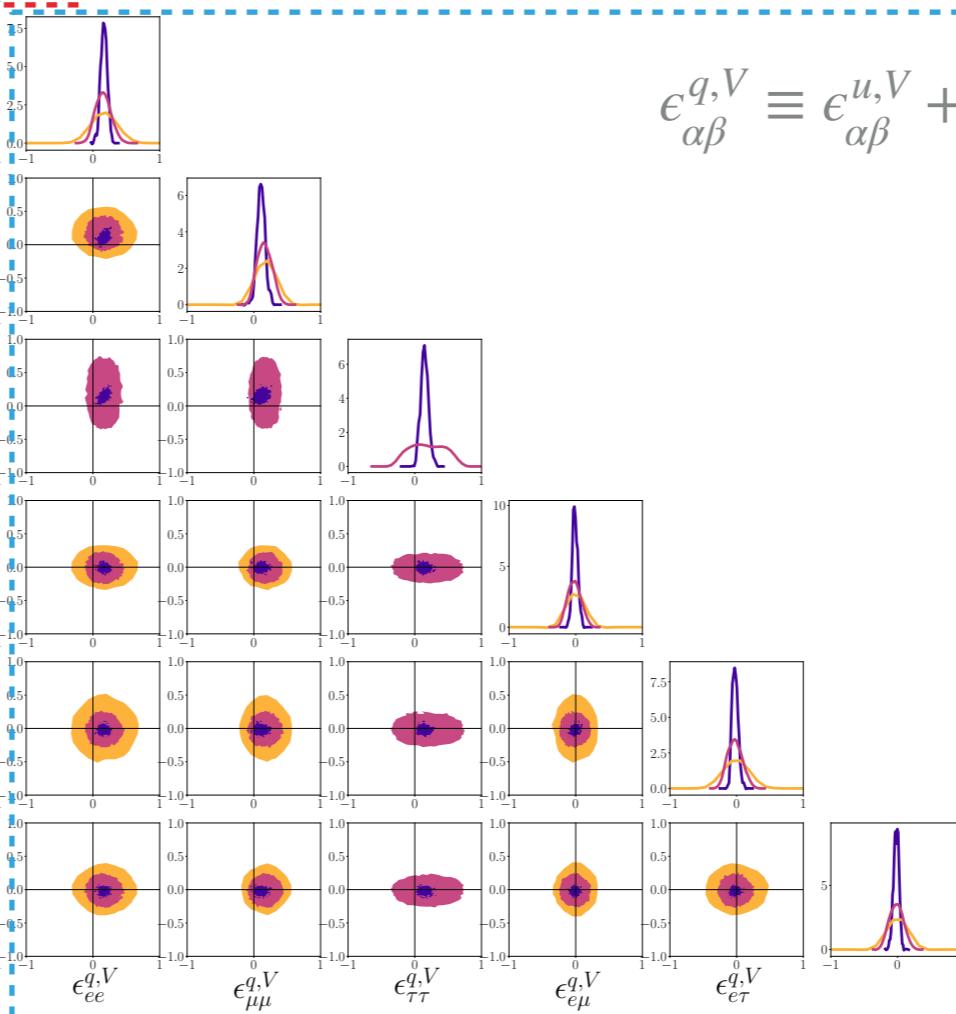


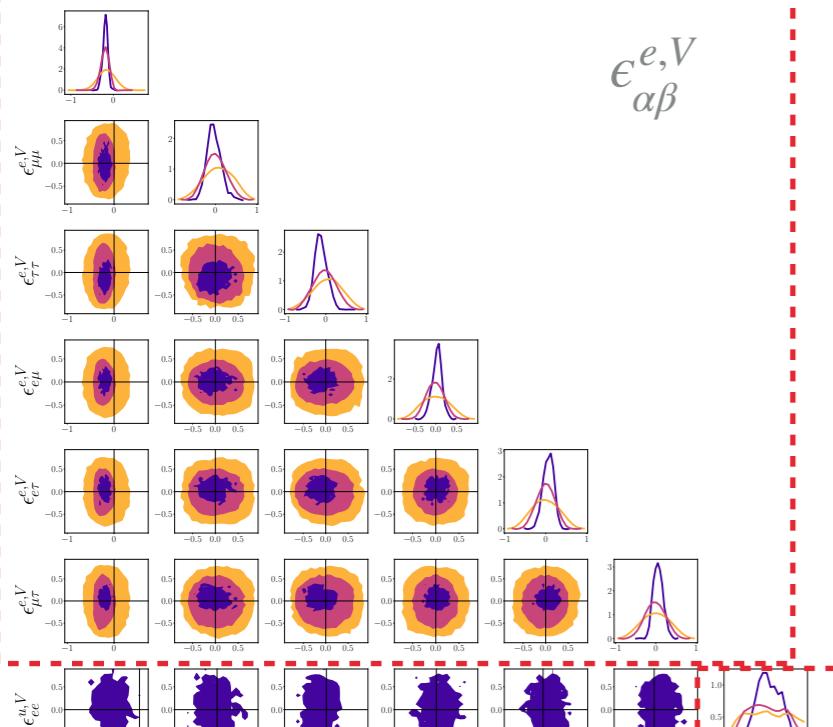
DUNE with priors from LXe

LXe atmospheric + solar with priors from COHERENT and Borexino

COHERENT CsI and Borexino

$$\epsilon_{\alpha\beta}^{q,V} \equiv \epsilon_{\alpha\beta}^{u,V} + \epsilon_{\alpha\beta}^{d,V}$$



$\epsilon_{\alpha\beta}^{e,V}$ 


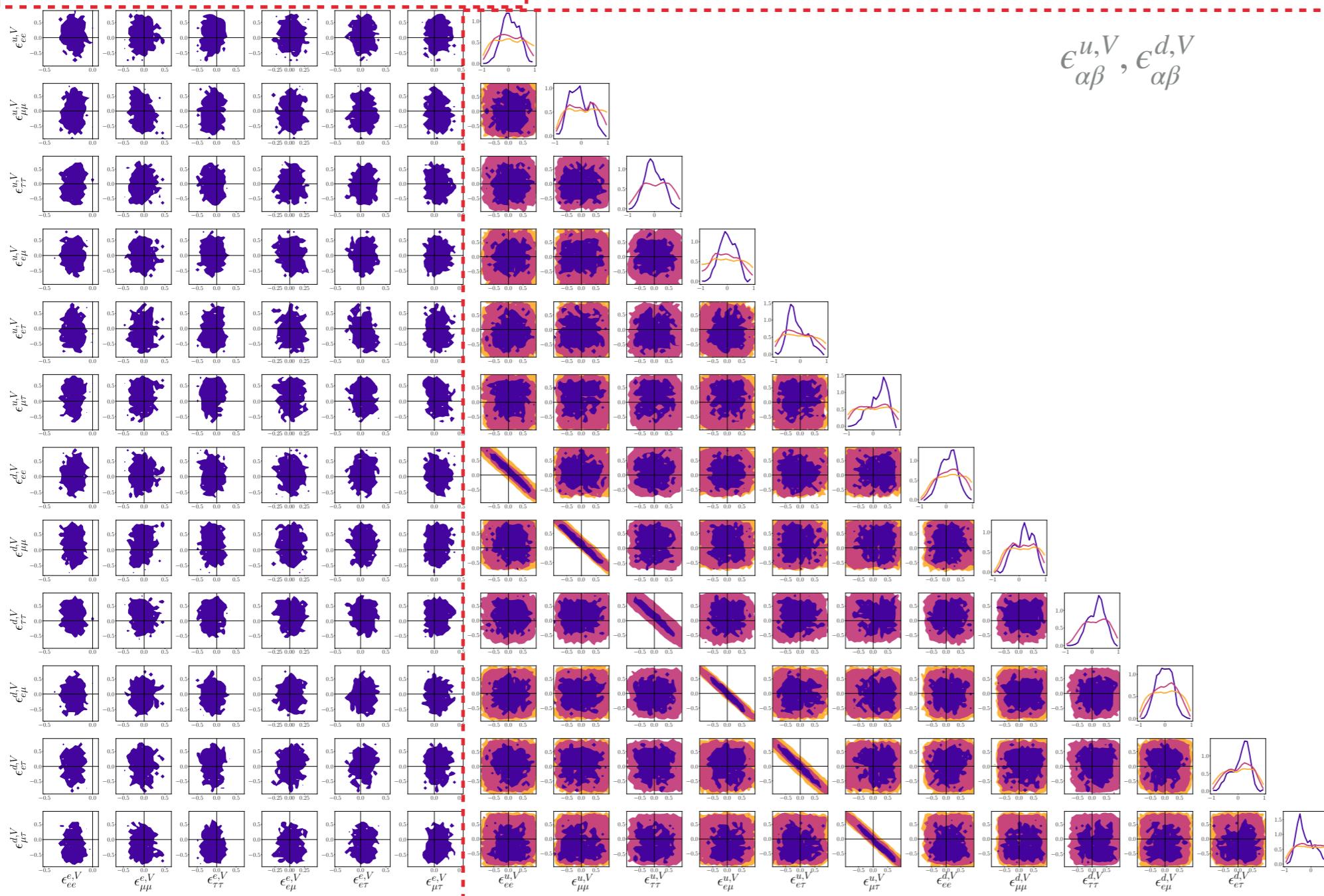
DUNE with priors from LXe



LXe atmospheric + solar with priors  
from COHERENT and Borexino



COHERENT CsI and Borexino

 $\epsilon_{\alpha\beta}^{u,V}, \epsilon_{\alpha\beta}^{d,V}$ 


## SUMMARY

- ▶ By using a prior-flow, we break up the computation cost of a global analysis into more manageable parts, utilizing copulas
- ▶ See 2002.03066 for more detail and NSI physics fit results
  - ▶ JHEP version coming soon
  - ▶ Copula code will soon be publicly available on GitHub

# BACKUP

$$dN = \sum_{\alpha, \beta} \frac{\mathcal{E}}{M_T} P_{\beta\alpha} \frac{d\Phi_\alpha}{dE_\nu} \frac{d\sigma_{\beta\alpha}(E_r, E_\nu)}{dE_r} dE_r dE_\nu$$

Computed with [pyCEvNS](#)

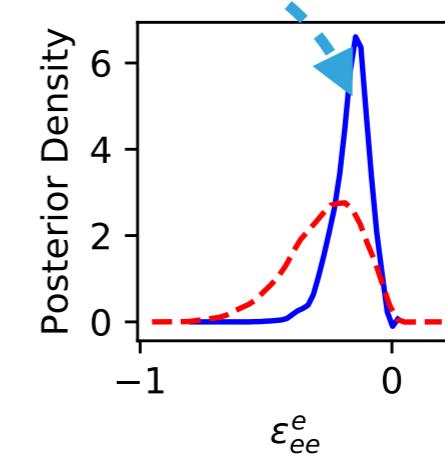
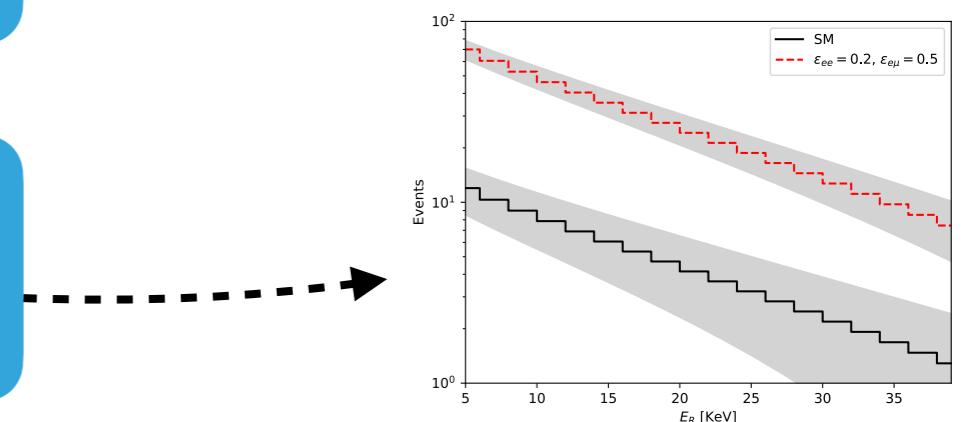
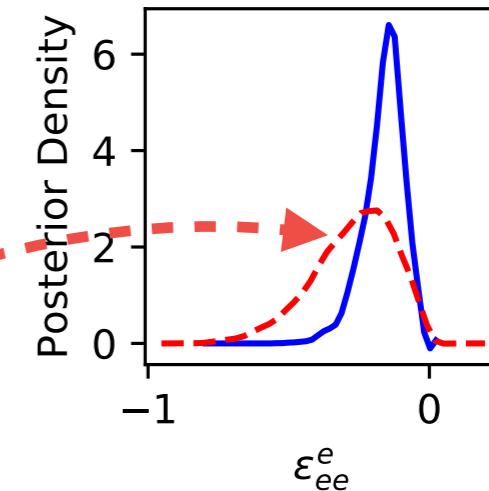
$$f(\vec{\epsilon}) = \frac{\mathcal{L}(\mathcal{D} | \vec{\epsilon}; H) \Pi(\vec{\epsilon})}{Z}$$

**MultiNest**  
[\(https://arxiv.org/abs/0809.3437\)](https://arxiv.org/abs/0809.3437)

Draw from Prior Distribution on  $\epsilon$

Simulate model prediction due to  $\epsilon$

Calculate Probability that  $\epsilon$  explains the data



## EXAMPLE: SIMULATING A GAUSSIAN COPULA

$$C_P^{Gauss}(\mathbf{u}) := \Phi_{\mathbf{P}} (\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_d))$$

Let  $P$  be a correlation matrix and compute the Cholesky decomposition  $P = A^T A$

1. Generate  $(Z_1, \dots, Z_d) \sim (N(0,1), \dots, N(0,1))$

Normal variates

2.  $\mathbf{X} = A^T \mathbf{Z}$  Correlate the variates!

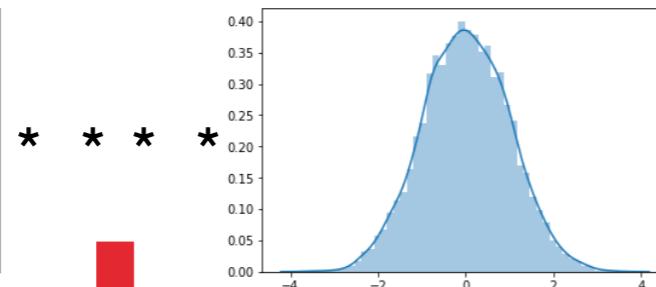
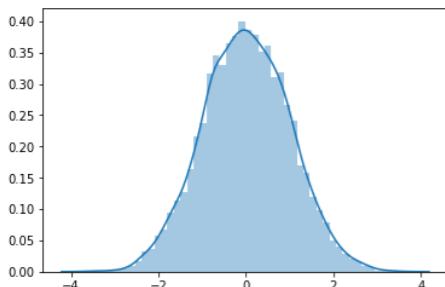
3.  $\mathbf{Y} = (\Phi(X_1), \dots, \Phi(X_d))$  Return them to  $[0,1]$

4. Return  $(u_1 = F_1^{-1}(Y_1), \dots, u_d = F_d^{-1}(Y_d))$

Match them up with our marginals

## EXAMPLE: GAUSSIAN COPULA

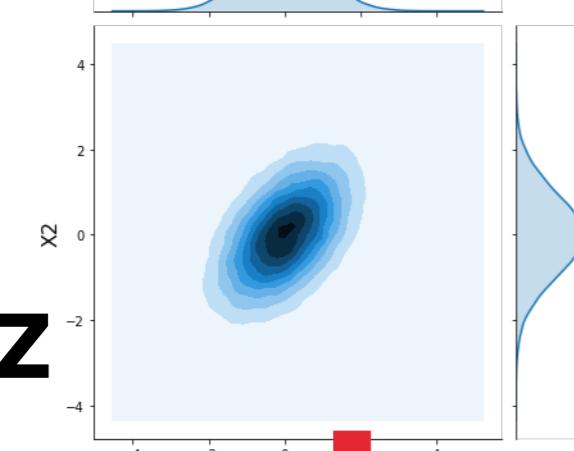
$$C_P^{Gauss}(\mathbf{u}) := \Phi_{\mathbf{P}} (\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_d))$$



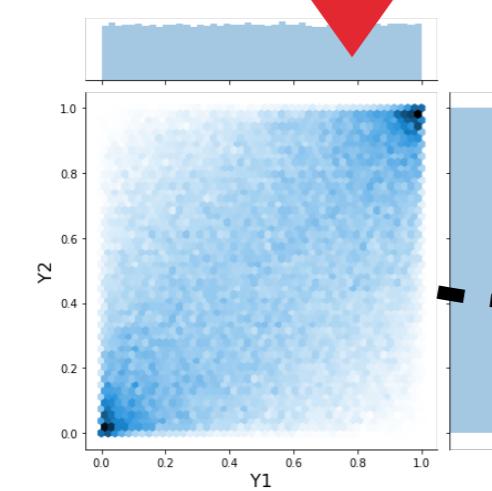
\* \* \* \*



$\mathbf{Z} \sim N(0,1)$

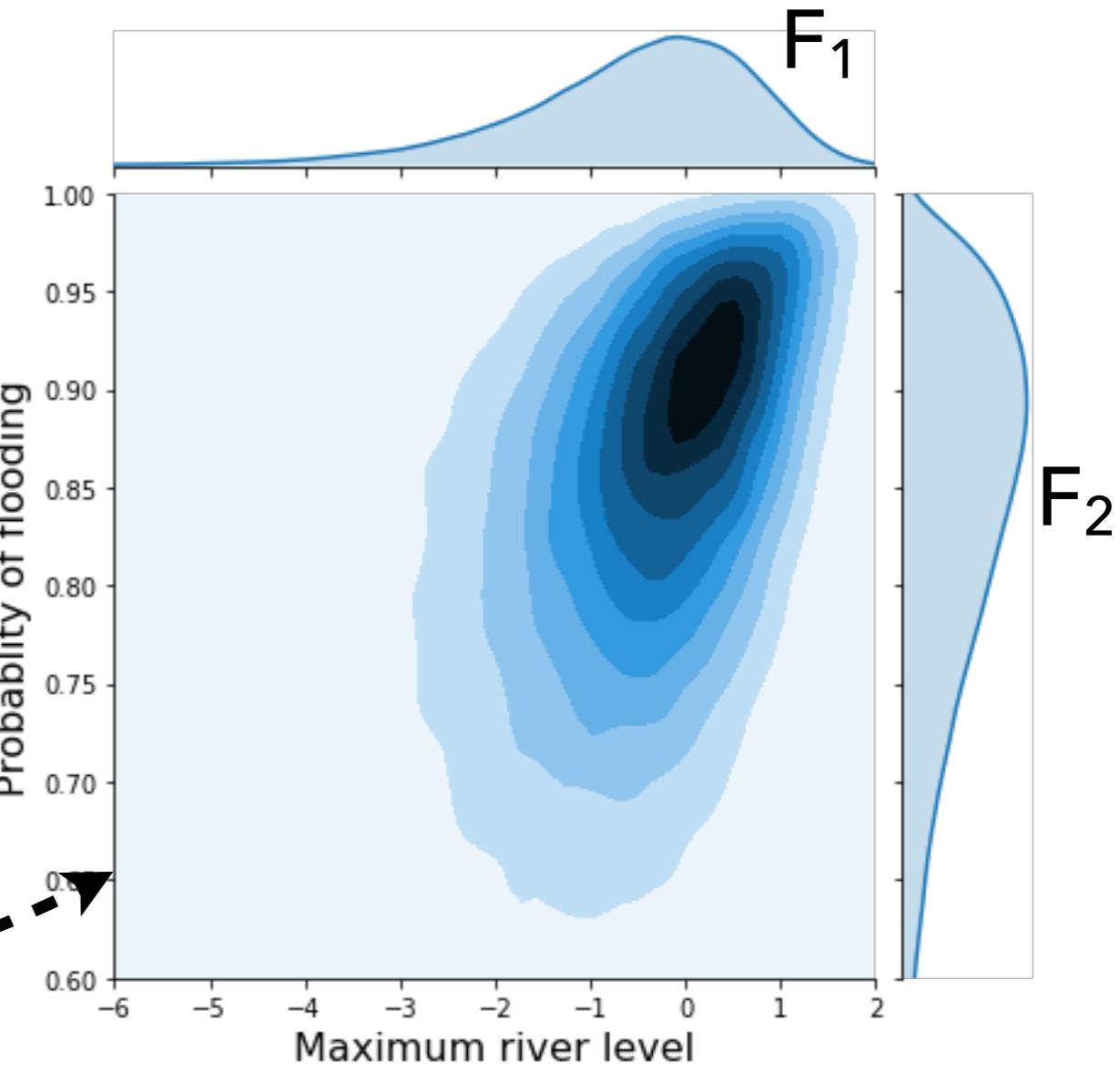


$\mathbf{X} = \mathbf{A}^T \mathbf{Z}$



$\mathbf{Y}$

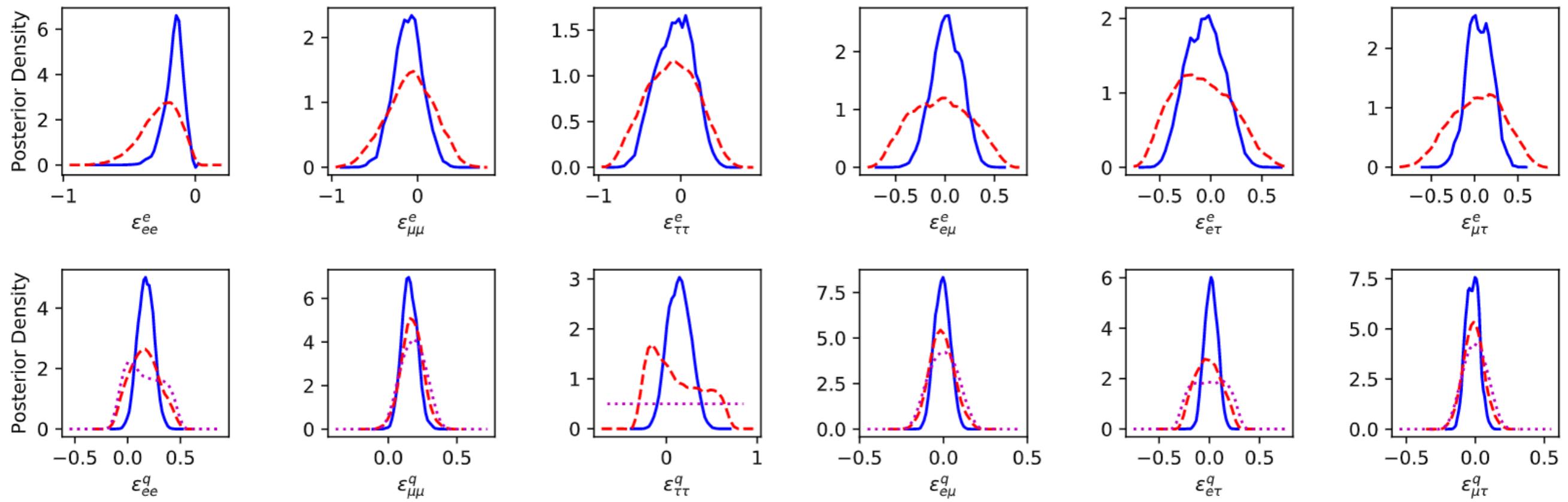
$F_1^{-1}, F_2^{-1}$



$F_1$

$F_2$

PRELIMINARY



COHERENT (Prior 1)

LXe (Prior 2)

DUNE (Final Posterior)

Final stage of the prior flow:

d=18, MultiNest completes in ~7000 cpu-hours

Compare to single-likelihood global analysis: Fails to converge in under 21000 cpu-hours

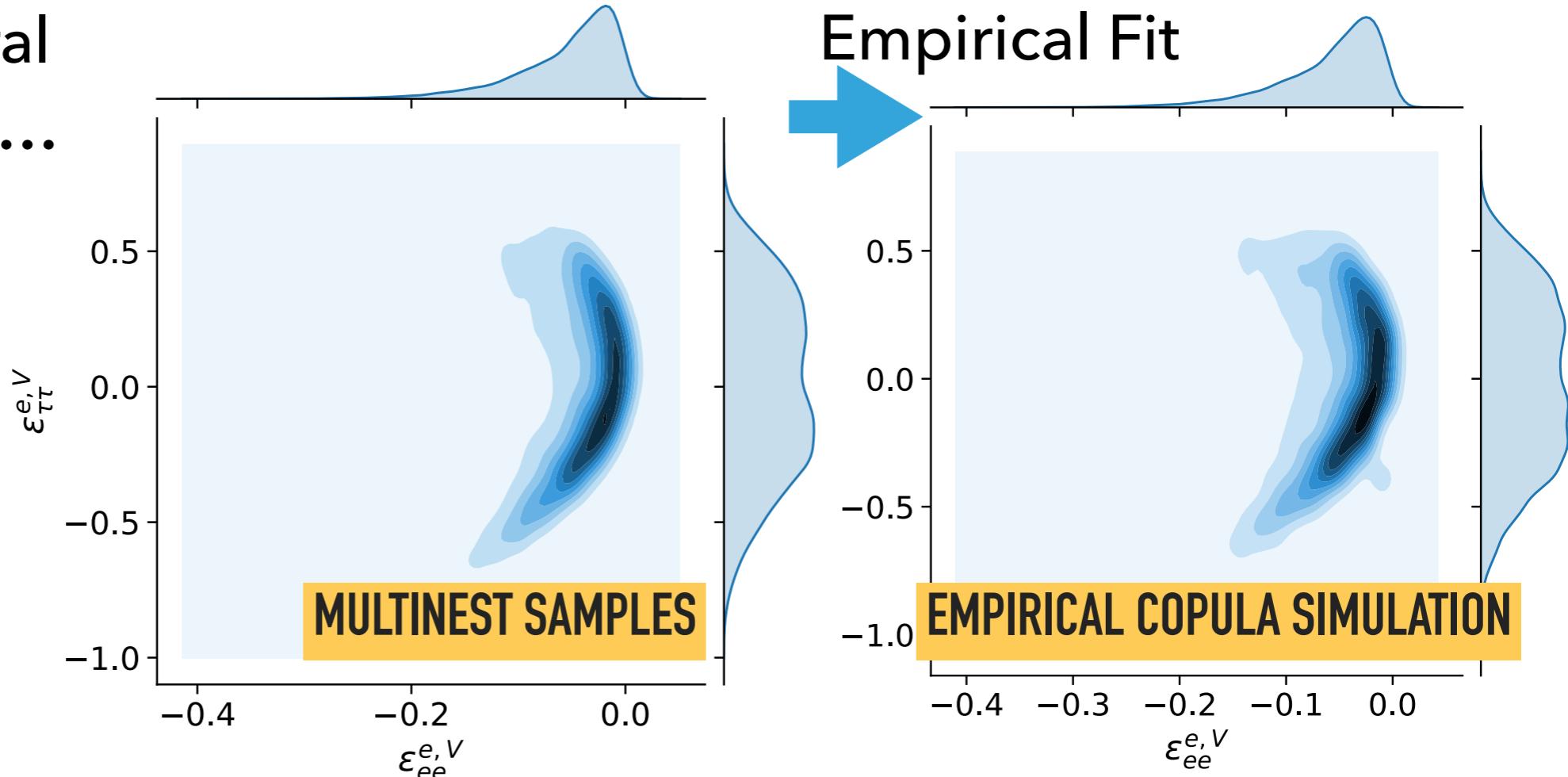
```

# Simulate bivariate pairs empirically.
class EmpiricalCopula:
    def __init__(self, datastr, i, j):
        # read in table
        robjects.r('data = read.table(file = "{0}", header=F)'.format(datastr))
        robjects.r('z = pobs(as.matrix(cbind(data[,{0}],data[,{1}]))))'.format(i, j))
    def simulate(self, u, v):
        def ddv(v2):
            v2 = float(v2)
            robjects.r('u = matrix(c({0}, {1}), 1, 2)'.format(u, v2))
            return np.asarray(robjects.r('dCn(u, U = z, j.ind = 1)'))
        try:
            return float(pynv.inversefunc(ddv, y_values=v, domain=[0, 1], open_domain=[True, True]))
        except:
            print("passing...")
            return v

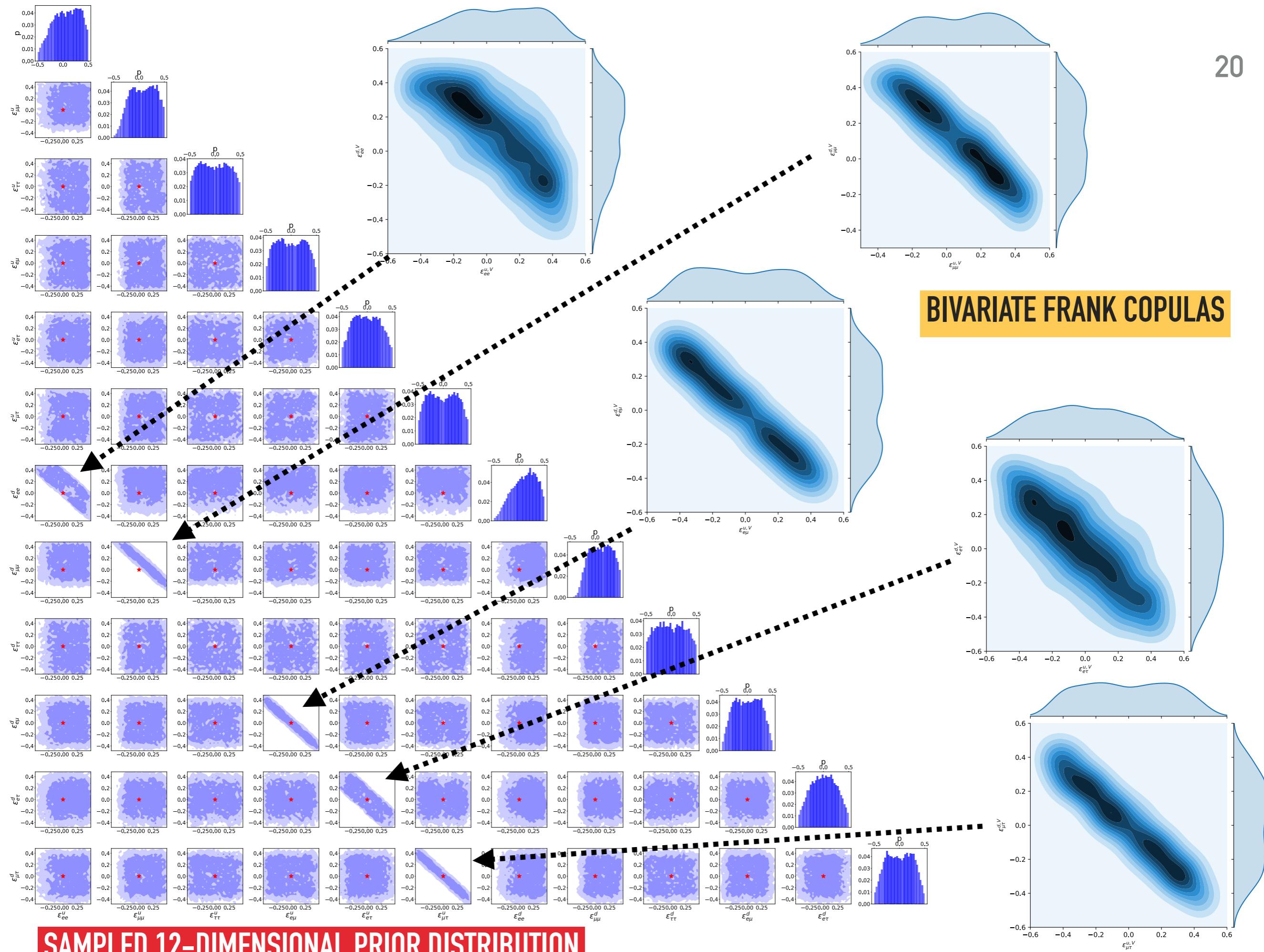
```

One can use several  
Copula libraries in...

- ▶ R
- ▶ Python
- ▶ Matlab



## BIVARIATE FRANK COPULAS



## Copula References:

- An *intuitive, visual guide to copulas*: <https://twiecki.io/blog/2018/05/03/copulas/>
- An *Introduction to Copulas*, Martin Haugh <http://www.columbia.edu/~mh2078/QRM/Copulas.pdf>
- *MULTINEST: an efficient and robust Bayesian inference tool for cosmology and particle physics*  
<https://arxiv.org/pdf/0809.3437.pdf>
- Li, 1999: [On Default Correlation: A Copula Function Approach](#)
- Somnath Chatterjee, Modelling credit risk - Bank of England 2015